# Rewriting OWL 2 QL Ontology-Mediated Queries: Succinctness

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$$\forall x, y\, \big(\varrho(x,y) \to \varrho'(x,y)\big) \qquad \forall x, y\, \big(\varrho(x,y) \land \varrho'(x,y) \to \bot\big)$$

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a **conjunctive query** (CQ) $q(\vec{x})$ is $\exists \vec{y}\, \varphi(\vec{x}, \vec{y})$,
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$\mathcal{C}_{\mathcal{T},\mathcal{A}}$ is the **canonical model** (**chase**) of $(\mathcal{T},\mathcal{A})$ and $\mathcal{C}_{\mathcal{T}}^{\tau(a)} = \mathcal{C}_{\mathcal{T},\{\tau(a)\}}$

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**UCQ** = unions of SPJ queries

as produced, e.g., by PerfectRef size $|q|^{|\mathcal{T}|} \cdot 2^{O(|q|^2)}$

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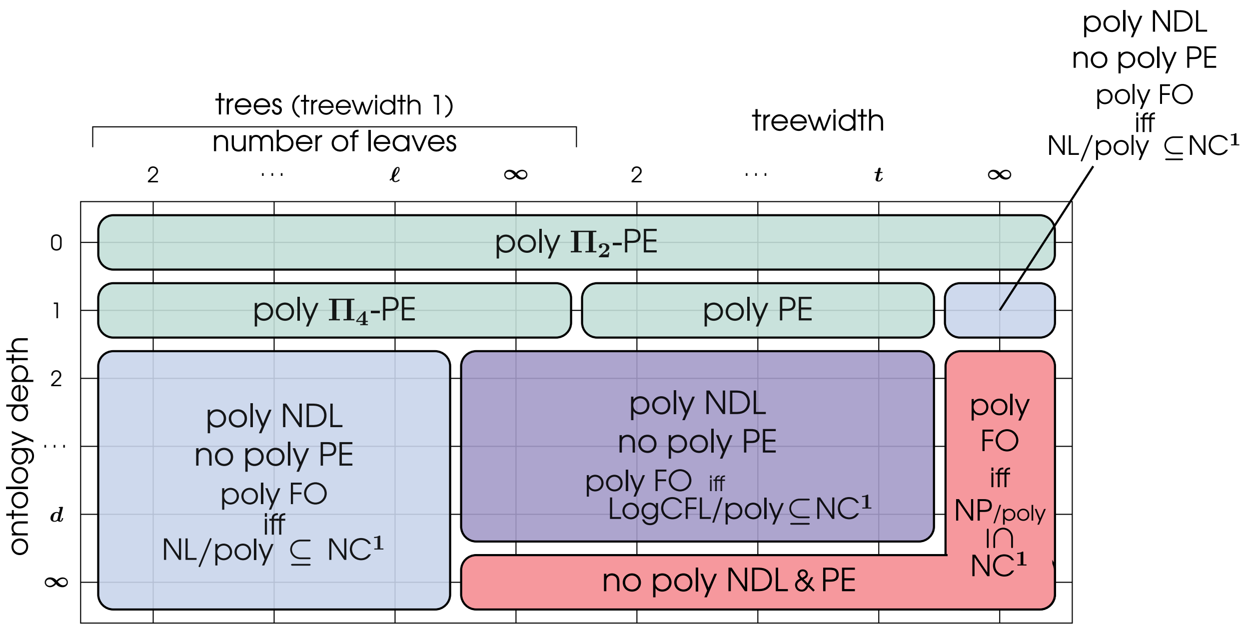
**no additional constants, no assumptions on data**

# Succinctness Landscape




ontology depth

$0$ = no axioms with $\exists y$ on the right-hand side

$d$ $\approx$ trees $\mathcal{C}_{\mathcal{T}}^{\tau(a)}$ of labelled nulls are of depth at most $d$

# Hypergraphs for OMQs

$A_1(x) \to \exists y \big(R(x,y) \land Q(y,x)\big)$

$A_1(x) \to \exists y\, P_\zeta(x,y)$ $\quad P_\zeta(x,y) \to R(x,y)$ $\quad P_\zeta(x,y) \to Q(y,x)$

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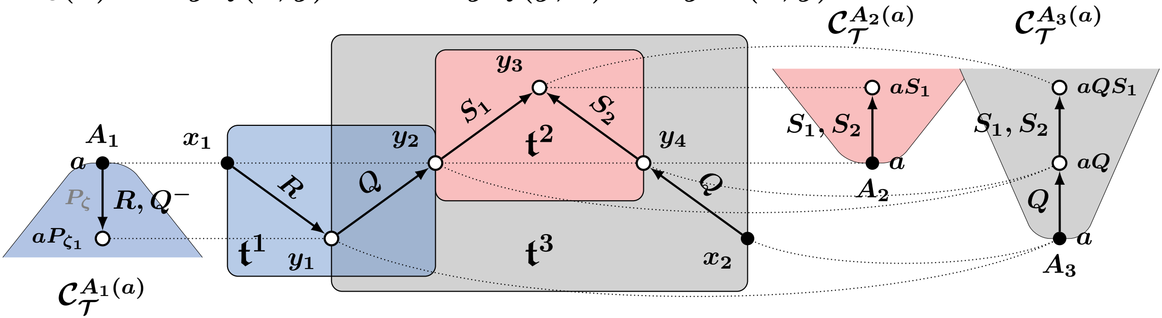

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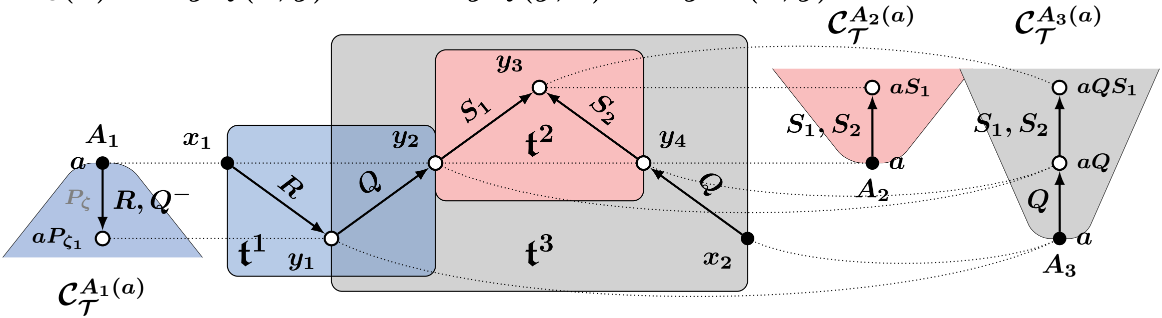
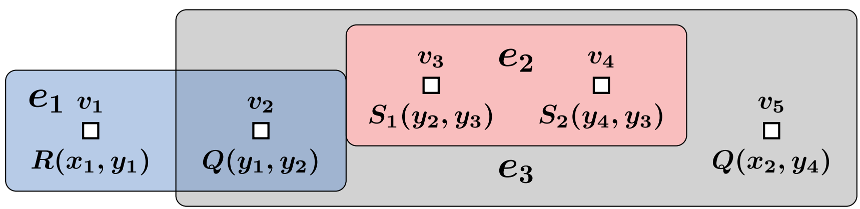



**hypergraph** $\mathcal{H}(Q)$ query atoms = vertices
sets of query atoms that can be mapped to trees = hyperedges



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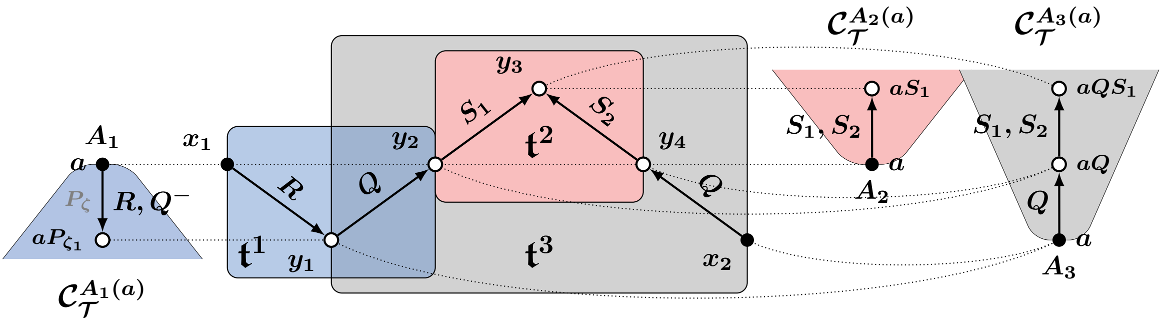


a **homomorphism** $q \to \mathcal{C}_{\mathcal{T},\mathcal{A}}$ = a map from variables of $q$ to **ind**$(\mathcal{A})$ + an **independent subset** of $\mathcal{H}(Q)$ such that

1. each hyperedge is 'generated' by the data
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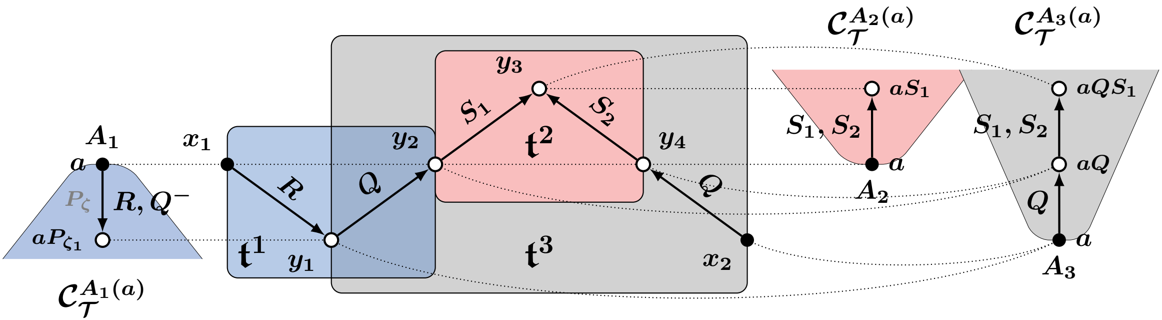


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**hypergraph function**

$$f_H = \bigvee_{E' \text{ independent}} \Big( \bigwedge_{v \in V \setminus V_{E'}} p_v \; \wedge \; \bigwedge_{e \in E'} p_e \Big)$$

# From OMQs to Hypergraph Programs (HGPs)

a **HGP** $P$ is a hypergraph $H$ whose vertices are labelled by $0$, $1$, or a literal over $p_1, \dots, p_n$

$P$ returns $1$ on an assignment $\alpha\colon \{p_1, \dots, p_n\} \to \{0, 1\}$ if there is an independent subset in $H$ that covers all zeroes under $\alpha$

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| OMQ $Q = (\mathcal{T}, q)$ | monotone HGP | of size | computes |
|---|---|---|---|
| $\mathcal{T}$ of depth 1 | HGP of degree 2 | $O(\lvert q\rvert)$ | $f_Q^\triangledown$ |
| tree-shaped $q$ with $\ell$ leaves | linear THGP | $\lvert q\rvert^{O(\ell^2)}$ | $f_Q^\triangledown$ |
| $q$ of treewidth $t$ and $\mathcal{T}$ of depth $d$ | THGP | $\lvert q\rvert^{O(1)} \cdot \lvert 2^{O(dt)}$ | $f_Q^\blacktriangledown$ |
| $q$ of treewidth $t$ and $\mathcal{T}$ of depth 1 | THGP of degree $2^{O(t)}$ | $\lvert q\rvert^{O(1)} \cdot 2^{O(t)}$ | $f_Q^\triangledown$ |
| tree-shaped $q$ and $\mathcal{T}$ of depth 1 | THGP of degree $2$ | $\lvert q\rvert^{O(1)}$ | $f_Q^\triangledown$ |

$f_Q^\triangledown = f_{\mathcal{H}(Q)}$ and $f_Q^\blacktriangledown$ is its modification for exponential $\mathcal{H}(Q)$

# Complexity of Boolean Functions v Size of Rewritings

**Theorem** If either $f_Q^{\triangledown}$ or $f_Q^{\blacktriangledown}$ is in

- $\mathsf{NC}^1$, then $Q$ has a polynomial FO-rewriting — Boolean formulas
- mP/poly, then $Q$ has a polynomial NDL-rewriting — monotone Boolean circuits
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Hint: on a single-object ABox, quantifiers are 'meaningless'
and the rewriting boils down to a propositional f-la, etc.

# Representation Results for Hypergraphs

**Theorem** (*i*) Any hypergraph $\boldsymbol{H}$ is isomorphic to
a subgraph of $\mathcal{H}(\boldsymbol{Q_H})$ for a polynomial-size $\boldsymbol{Q_H}$ with an ontology of **depth 2**

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**Theorem** Any **tree** hypergraph $H$ with $\ell$ leaves is isomorphic to
a subgraph of $\mathcal{H}(T_H)$ for a polynomial-size $T_H$ with an of ontology of **depth 2**
and a **tree-shaped CQ** with $\ell$ leaves
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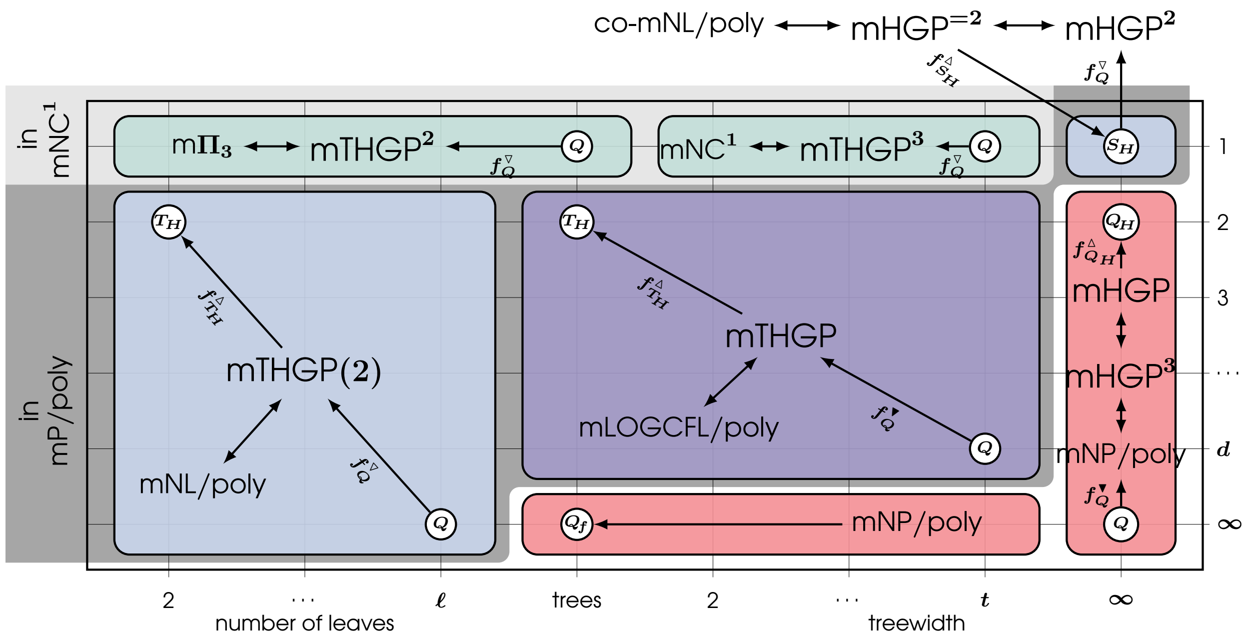
# Roadmap for Succinctness Proofs




$$\mathsf{m}\Pi_3 \subsetneq \mathsf{mAC}^0 \subsetneq \mathsf{mNC}^1 \subsetneq \mathsf{mNL/poly} \subseteq \mathsf{mLOGCFL/poly} \subsetneq \mathsf{mP/poly} \subsetneq \mathsf{mNP/poly}$$

**Boolean formulas** **nondeterministic Boolean circuits**

# Conclusions

- HGPs provide a natural link between complexity classes and hypergraph functions

- polynomial **PE-rewritings** exists only in for very **restricted** cases of ontologies of depth 0 & 1 (except unbounded treewidth)

- polynomial **NDL-rewritings** exists for most cases where query answering is **tractable**

optimal NDL-rewritings in Stas's talk

- existence of polynomial PE-, NDL- and FO-rewritings is closely related to **circuit complexity**