ON THE ASSERTIONAL LOGICS OF THE GENERIC POINTED DISCRIMINATOR AND GENERIC POINTED FIXEDPOINT DISCRIMINATOR VARIETIES

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Dedicated to the memory of Willem Blok

ABSTRACT. The *generic pointed discriminator variety* is the variety generated by the class of all algebras $\langle A; t, 1 \rangle$ where t is the ternary discriminator on A and 1 is a nullary operation. The *generic pointed fixedpoint discriminator variety* is defined similarly.

A deductive system is said to be a *pointed* (*fixedpoint*) *discriminator logic* if it is algebraisable and its largest equivalent algebraic semantics is generated as a quasivariety by a class of pointed (fixedpoint) discriminator algebras. Examples of pointed (fixedpoint) discriminator logics abound in the literature and include classical propositional logic; the modal logic **S5**; the n-dimensional cylindric logics; the n-valued Post logics; the n-valued Łukasiewicz logics; and the various algebraisable implicational fragments of these deductive systems.

From simple and well known results about (fixedpoint) discriminator varieties, it follows that every pointed (fixedpoint) discriminator variety is term equivalent to a subvariety of the generic pointed (fixedpoint) discriminator variety with additional operations that are compatible with the generic pointed (fixedpoint) discriminator congruences in a natural way. From logical analogues of these results, it follows that every pointed (fixedpoint) discriminator logic is definitionally equivalent to an axiomatic extension of the expansion of the assertional logic of the generic pointed (fixedpoint) discriminator variety by extensional logical connectives.

In this paper, the assertional logics inherent in the generic pointed discriminator variety and the generic pointed fixedpoint discriminator variety are investigated. Two non-Fregean deductive systems, called *BCSK logic* and the *skew Boolean propositional calculus*, are introduced; these closely resemble the purely implicational fragment of classical propositional logic and the negation-free fragment of the classical propositional calculus, respectively. The two main results of the paper show that every pointed discriminator logic [resp. pointed fixedpoint discriminator logic] is definitionally equivalent to an axiomatic extension of the expansion of the skew Boolean propositional calculus [resp. BCSK logic] by extensional logical connectives.

The results provide a simple and uniform description of many deductive systems previously considered in the literature, and throw new light on the structure of logics arising from pointed (fixedpoint) discriminator varieties.

1. INTRODUCTION

About this paper. One of the projects of abstract algebraic logic reverses the traditional process of the algebraisation of logic by considering various families of quasivarieties, which have not originally arisen via the process of algebraisation, and investigating the logical systems that may be inherent in them [23, p. 546]. In its most general form, this program asks [58, p. 82]: Is there a characterisation