Monodic fragments of first-order temporal logics: 2000–2001 A.D.

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Abstract. The aim of this paper is to summarize and analyze some results obtained in 2000–2001 about decidable and undecidable fragments of various first-order temporal logics, give some applications in the field of knowledge representation and reasoning, and attract the attention of the 'temporal community' to a number of interesting open problems.

1 Introduction

Temporal logic has found numerous applications in computer science, ranging from the traditional and well-developed fields of program specification and verification [45, 42, 43], temporal databases [20, 21, 3, 50, 30], and distributed and multi-agent systems [27], to more recent uses in knowledge representation and reasoning [6, 7, 10, 49, 60]. This is true of both propositional and first-order temporal logic. However, the mainstream of theoretical studies in the discipline has mostly been restricted to the propositional case—witness the surveys [26, 51], or the two-volume monograph [28, 30] where only one chapter is devoted to first-order temporal logics.

The reason for this seems clear. Though some axiomatizations of first-order temporal logics are known (e.g., [48] presents axiomatizations for first-order logics with Until and Since over the class of all linear flows and over the rationals), a series of incompleteness theorems [1, 4, 28, 31, 44, 53, 54], started by unpublished results of Scott and Lindström in the 1960s, show that many of the first-order temporal logics most useful in computer science are not even recursively enumerable. But in contrast to classical first-order logic, where the early undecidability results of Turing and Church stimulated research and led to a rich and profound theory concerned with classifying fragments of first-order logic according to their decidability (see, e.g., [13]), for a long time there were few if any serious attempts to convert the 'negative' results in first-order temporal logic into a classification problem. Apparently, the extremely weak expressive power of the temporal formulas required to prove undecidability left no hope that any useful decidable fragments located 'between' propositional and first-order temporal logics could ever be found.

A certain breakthrough has been recently achieved in [35, 61], where the so-called *monodic fragment* of first-order temporal and modal logics was shown to have much better computational behavior. In monodic formulas, the scope of temporal operators is restricted only to subformulas with at most one free variable. The idea to consider them came from knowledge representation, where a number of decidable description logics with modal, temporal, dynamic, and similar operators were constructed [49, 9, 56, 60]. It turned out that if these operators are applicable only to concepts (i.e., unary predicates) or subsumptions (closed formulas) of a decidable description logic, then the resulting hybrid logic is usually decidable as well; but applications of temporal operators to roles (i.e., binary predicates) lead to undecidable logics [57].

The aim of this paper is to survey the rapidly growing body of results, obtained since 2000, about decidable and undecidable fragments of various first-order temporal logics. We will describe some applications in knowledge representation and reasoning. We also include some interesting open problems, which we hope will attract the attention of the 'temporal community' in both logic and computer science.

2 First-order logics of linear and branching time

2.1 Linear time

Denote by $\mathcal{QTL}$ the *first-order temporal language* constructed in the standard way from the following alphabet:

- *predicate symbols* $P_0, P_1, \dots$, each of which is of some fixed arity,
- *individual variables* $x_0, x_1, \dots$,
- *individual constants* $c_0, c_1, \dots$,
- the *booleans* $\wedge$, $\neg$,
- the *universal quantifier* $\forall x$ for each individual variable x,
- the *temporal operators* $\mathcal{S}$ (Since) and $\mathcal{U}$ (Until).

Note that the language contains neither equality nor function symbols.

We will use the following standard abbreviations:

$$\exists x\varphi = \neg\forall x\neg\varphi \qquad \Diamond_F\varphi = \top\mathcal{U}\varphi$$
$$\Box_F\varphi = \neg\Diamond_F\neg\varphi \qquad \Box_F^+\varphi = \varphi \wedge \Box_F\varphi$$
$$\Diamond_F^+\varphi = \varphi \vee \Diamond_F\varphi \qquad \bigcirc\varphi = \bot\mathcal{U}\varphi$$

where $\top$ and $\bot$ are the boolean constants 'truth' and 'falsehood', respectively. Thus, $\Diamond_F$ can be read as 'some time in the future', $\Box_F^+$ as 'from now on', and $\bigcirc$ as 'at the next moment' or 'tomorrow'.

$\mathcal{QTL}$ is interpreted in *first-order temporal models* of the form $\mathfrak{M} = \langle \mathfrak{F}, D, I \rangle$, where $\mathfrak{F} = \langle W, < \rangle$ is a strict linear order representing the intended *flow of time*,

D is a non-empty set, the *domain* of $\mathfrak{M}$, and I is a function associating with every moment of time $w \in W$ a first-order structure

$$I(w) = \left\langle D, P_0^{I(w)}, \dots, c_0^{I(w)}, \dots \right\rangle,$$

the *state* of $\mathfrak{M}$ at moment w. Here, for each i, $P_i^{I(w)}$ is a predicate on D of the same arity as P_i (for a propositional variable p_i, the predicate $p_i^{I(w)}$ is simply true or false), and $c_i^{I(w)}$ is an element of D. We require that $c_i^{I(w)} = c_i^{I(v)}$ for any $w, v \in W$—i.e., that constants are 'rigid'.

An *assignment in* D is a function $\mathfrak{a}$ from the set *var* of individual variables to D. We extend $\mathfrak{a}$ to constants via $\mathfrak{a}(c) = c^{I(w)}$ for any $w \in W$; this is well-defined. The *truth-relation* $(\mathfrak{M}, w) \models^{\mathfrak{a}} \varphi$ (or simply $w \models^{\mathfrak{a}} \varphi$, if $\mathfrak{M}$ is understood) in the model $\mathfrak{M}$ under the assignment $\mathfrak{a}$ is now defined inductively in the usual way:

- $w \models^{\mathfrak{a}} P_i(y_1, \dots, y_\ell)$ iff $P_i^{I(w)}(\mathfrak{a}(y_1), \dots, \mathfrak{a}(y_\ell))$ is true in $I(w)$, where the y_i are variables or constants;
- $w \models^{\mathfrak{a}} \varphi \wedge \psi$ iff $w \models^{\mathfrak{a}} \varphi$ and $w \models^{\mathfrak{a}} \psi$;
- $w \models^{\mathfrak{a}} \neg\psi$ iff $w \not\models^{\mathfrak{a}} \psi$;
- $w \models^{\mathfrak{a}} \forall x \psi$ iff $w \models^{\mathfrak{b}} \psi$ for every assignment $\mathfrak{b}$ in D that may differ from $\mathfrak{a}$ only on x;
- $w \models^{\mathfrak{a}} \varphi \mathcal{S} \psi$ iff there is $v < w$ such that $v \models^{\mathfrak{a}} \psi$ and $u \models^{\mathfrak{a}} \varphi$ for every u in the interval $(v, w) = \{u \in W : v < u < w\}$;
- $w \models^{\mathfrak{a}} \varphi \mathcal{U} \psi$ iff there is $v > w$ such that $v \models^{\mathfrak{a}} \psi$ and $u \models^{\mathfrak{a}} \varphi$ for every $u \in (w, v)$.

For a class $\mathcal{F}$ of strict linear orders, we let $QTL(\mathcal{F})$, 'the *temporal logic of* $\mathcal{F}$', denote the set of $\mathcal{QTL}$-formulas that are true at all times in all models based on flows of time in $\mathcal{F}$ under all assignments. $QTL_{fin}(\mathcal{F})$ consists of the $\mathcal{QTL}$-formulas that are valid in all models based on flows of time in $\mathcal{F}$ and having *finite domains*.

Remark 1. In this paper we consider only models with *constant domains*. Satisfiability in models with expanding domains is known to be reducible to satisfiability in models with constant domains; see, e.g., [61].

2.2 Branching time

There are a number of approaches to constructing temporal logics based on the branching time paradigm; see, e.g., [16, 26, 64, 30]. Many of the resulting languages turn out to be fragments of the language we call here $\mathcal{QPCTL}^*$, *quantified* $\mathcal{CTL}^*$ *with past operators*. It is obtained by extending the vocabulary of $\mathcal{QTL}$ with a unary *path quantifier* A, which can be applied to arbitrary formulas. We let $\mathsf{E}\varphi$ abbreviate $\neg\mathsf{A}\neg\varphi$.

Important sublanguages of $\mathcal{QPCTL}^*$ are:

- $\mathcal{QCTL}^*$, that is $\mathcal{QPCTL}^*$ without the past operator $\mathcal{S}$;

– $\mathcal{QCTL}^*_F$, that is $\mathcal{QCTL}^*$ in which the binary operator $\mathcal{U}$ is replaced with the unary operator $\Box_F$ 'always in the future';
– $\mathcal{QPCTL}$, the fragment of $\mathcal{QPCTL}^*$ in which the path quantifiers and temporal operators occur only in the form $\mathsf{E}(\psi_1\mathcal{U}\psi_2)$, $\mathsf{E}(\psi_1\mathcal{S}\psi_2)$, $\mathsf{A}(\psi_1\mathcal{U}\psi_2)$, or $\mathsf{A}(\psi_1\mathcal{S}\psi_2)$.

A *tree* is a strict partial order $\mathfrak{F} = \langle W, < \rangle$ such that for all $w \in W$, the set $\{v \in W : v < w\}$ is well-ordered by $<$. A *full branch* of $\mathfrak{F}$ is a maximal linearly-ordered subset of W. An ω*-tree* is a tree whose full branches, ordered by $<$, are all order-isomorphic to $\langle \mathbb{N}, < \rangle$. $\mathcal{QPCTL}^*$ (as well as its sublanguages) is interpreted in structures of the form $\mathfrak{M} = \langle \mathfrak{F}, \mathcal{H}, D, I \rangle$, where $\mathfrak{F} = \langle W, < \rangle$ is an ω-tree, $\mathcal{H}$ is a set (*bundle*) of full branches in $\mathfrak{F}$ with $\bigcup \mathcal{H} = W$, D is a non-empty set called the *domain* of $\mathfrak{M}$, and I is again a function associating with every $w \in W$ a first-order structure $I(w)$ with domain D. Constants are again rigid. We call $\mathfrak{M}$ a *bundled tree model,* and the branches in the bundle $\mathcal{H}$ are called *histories.* In the case when $\mathcal{H}$ contains all full branches in $\mathfrak{F}$, we say that $\mathfrak{M}$ is a *full tree model,* or simply a *tree model.*

Assignments $\mathfrak{a}$ in D are defined as in the linear case. The *truth-relation* $(\mathfrak{M}, h, w) \models^{\mathfrak{a}} \varphi$, for $w \in h \in \mathcal{H}$ (or simply $(h, w) \models^{\mathfrak{a}} \varphi$ if $\mathfrak{M}$ is understood), is defined as follows:

– $(h, w) \models^{\mathfrak{a}} P_i(y_1, \dots, y_\ell)$ iff $P_i^{I(w)}(\mathfrak{a}(y_1), \dots, \mathfrak{a}(y_\ell))$ is true in $I(w)$, where the y_i are variables or constants;
– $(h, w) \models^{\mathfrak{a}} \forall x\psi$ iff $(h, w) \models^{\mathfrak{b}} \psi$ for every assignment $\mathfrak{b}$ in D that may differ from $\mathfrak{a}$ only on x;
– $(h, w) \models^{\mathfrak{a}} \varphi\mathcal{S}\psi$ iff there is $v < w$ such that $(h, v) \models^{\mathfrak{a}} \psi$ and $(h, u) \models^{\mathfrak{a}} \varphi$ for every $u \in (v, w)$;
– $(h, w) \models^{\mathfrak{a}} \varphi\mathcal{U}\psi$ iff there is $v \in h$ such that $v > w$, $(h, v) \models^{\mathfrak{a}} \psi$ and $(h, u) \models^{\mathfrak{a}} \varphi$ for every $u \in (w, v)$;
– $(h, w) \models^{\mathfrak{a}} \mathsf{A}\psi$ iff for all $h' \in \mathcal{H}$ such that $w \in h'$ we have $(h', w) \models^{\mathfrak{a}} \psi$,

plus the standard clauses for the booleans.

For any of the branching time languages $\mathcal{L}$ introduced above, denote by BL (L) the set of all $\mathcal{L}$-formulas that are true at all points in all histories under every assignment in every bundled (respectively, full) tree model. Thus, $BQPCTL^*$ is the set of $\mathcal{QPCTL}^*$-formulas valid in bundled tree models, while $QPCTL^*$ is the set of $\mathcal{QPCTL}^*$-formulas valid in all tree models.

Remark 2. The logic $BQPCTL^*$ is the first-order version of bundled Ockhamist logic; cf. e.g. [30]. In the computer science literature, $\mathcal{QCTL}^*$ often denotes only the set of *state* (i.e., history-independent) $\mathcal{QCTL}^*$*-formulas.* However, this makes no difference as far as decidability is concerned, because a *path* (history-dependent) *formula* φ is satisfiable iff the state formula $\mathsf{E}\varphi$ is satisfiable.

Satisfiability in bundled tree models can be reduced to satisfiability in full tree models. Indeed, given a $\mathcal{QPCTL}^*$-formula φ, let q be a propositional variable not occurring in φ, and denote by $\varphi^{\uparrow}$ the result of replacing each subformula of φ of the form $\mathsf{A}\psi$ by

$$\mathsf{A}(\Diamond_F \Box_F q \to \psi).$$

Then we have the following:

Proposition 1 ([36]). *φ is satisfiable in a bundled tree model iff $\mathsf{E}\Diamond_F\Box_F q \wedge \varphi^{\uparrow}$ is satisfiable in a full tree model.*

Example 1 in Section 3.2 illustrates the difference between satisfiability in bundled and full tree models.

3 Who needs decidable first-order temporal logics?

As was mentioned in the introduction, temporal logic has a wide range of applications in various fields of computer science and artificial intelligence. Here we give just two examples from knowledge representation in AI, where the standard requirements of KR formalisms are *effectiveness and practical implementability.* We show how to embed the KR systems into first-order temporal logic; this will be used in Section 7 to obtain decidability results.

3.1 Temporal description logics

Classical description logic is a family of KR formalisms intended for dealing with conceptual knowledge about *static* application domains (see, e.g., [18] and references therein). In a description logic, knowledge is represented by means of subsumption relations between complex concepts and instance assertions between objects and concepts. For example, in the logic $\mathcal{ALC}$ of [14], complex concepts are composed from a countably infinite set $A_1, A_2, \dots$ of atomic concepts (unary predicates) by means of the boolean operators $\sqcap$ and $\neg$ and the quantifiers $\forall R$ and $\exists R$, where R is a role name (binary predicate) from a countably infinite set of role names. Concepts are evaluated in $\mathcal{ALC}$-models

$$I = \langle \Delta, A_1^I, \dots, R_1^I, \dots, a_1^I, \dots \rangle,$$

where Δ is a non-empty set, the A_i^I are subsets of Δ, the R_i^I subsets of $\Delta \times \Delta$, and the a_i^I are members of Δ interpreting the object names a_i of $\mathcal{ALC}$. The value C^I of a complex concept C is defined inductively, the only non-trivial clause being the following:

– $(\exists R.C)^I = \{x \in \Delta : \exists y \in \Delta\ (xR^I y \wedge y \in C^I)\}$.

We say that C_1 *subsumes* C_2 in I, $I \models C_1 \sqsubseteq C_2$ in symbols, if $C_1^I \subseteq C_2^I$. We say that an object name a is an *instance* of C in I, $I \models a : C$ in symbols, if $a^I \in C^I$. For example, the subsumption

$$\mathsf{Person} \sqcap \exists \mathsf{lives\_in}.\mathsf{Bavaria} \sqsubseteq \mathsf{Person} \sqcap \forall \mathsf{drinks}.\mathsf{Beer}$$

says that every person living in Bavaria drinks only beer, while the assertion

$$John : \exists \mathsf{drinks}.\mathsf{Whiskey}$$

claims that John drinks whiskey (and possibly some other liquids).

To represent *dynamic* conceptual knowledge bases, description logics have been extended by means of temporal operators; see, e.g., [49, 9, 60, 7]. On the semantical level, such an extension is quite straightforward. Models for temporal description logics are obtained from first-order temporal models by taking I to be a function associating with every time point $w \in W$ not a first-order model but an $\mathcal{ALC}$-model $I(w) = \left\langle \Delta, A_1^{I(w)}, \dots, R_1^{I(w)}, \dots, a_1^{I(w)}, \dots \right\rangle$, where $a_i^{I(w)} = a_i^{I(v)}$ for all $v, w \in W$.

On the syntactic side, we can obtain a hierarchy of temporalized description logics by allowing applications of temporal operators to various syntactic entities of $\mathcal{ALC}$. We can apply them to concepts, as in

$$\mathsf{Ordered\_object} \sqsubseteq \bigcirc \bigcirc \mathsf{Delivered\_object}$$

('each ordered object will be delivered in two days'), or

$$\mathsf{Mortal} = \mathsf{Living\_being} \sqcap (\mathsf{Living\_being}\ \mathcal{U}\ \Box_F \neg \mathsf{Living\_being})$$

('mortals are living beings which eventually die'). We can also apply them to subsumption relations, as in

$$\Box_F \Diamond_F \neg(\mathsf{Living\_being} \sqsubseteq \bot)$$

('living beings will never die out completely'). If time is branching, we can also say

$$\mathsf{E}\Box_F(\mathsf{Ordered\_object} \sqsubseteq \bigcirc \mathsf{Delivered\_object})$$

('it is possible that every ordered object is always delivered in one day'), or

$$\mathsf{Ordered\_object} \sqsubseteq \mathsf{A} \bigcirc \mathsf{Delivered\_object}$$

('an ordered object must be delivered the next day').

The maximal language $\mathcal{DPCTL}^*$ ($\mathcal{Q}$ for 'quantified' is replaced by $\mathcal{D}$ for 'description') that we consider in this paper allows applications of the temporal operators and path quantifiers to both concepts and subsumption relations.[1] More precisely, if C and D are concepts then $C\mathcal{U}D$, $C\mathcal{S}D$, $\mathsf{E}C$ and $\mathsf{A}C$ are also concepts. Atomic $\mathcal{DPCTL}^*$-formulas are of the form $a : C$, aRb, and $C \sqsubseteq D$, where C, D are $\mathcal{DPCTL}^*$-concepts, R is a role name, and a, b are object names. Complex formulas are composed from atomic ones using the booleans, $\mathcal{S}$, $\mathcal{U}$, and A.

The semantics of $\mathcal{DPCTL}^*$ should be clear. First, given a bundled tree model $\mathfrak{M} = \langle \mathfrak{F}, \mathcal{H}, \Delta, I \rangle$, we define the extension $C^{I(h,w)}$ of a concept C relative to a history h and a moment w in h. This is done by induction as follows (we list only the interesting clauses):

[1] We could, of course, allow for temporalized roles as well. However, the obtained temporal description logics would be undecidable; see [60] for more information.

- $A^{I(h,w)} = A^{I(w)}$, for all concept names A;
- $(\exists R.C)^{I(h,w)} = \{x \in \Delta : \exists y\ (xR^{I(w)}y \wedge y \in C^{I(h,w)})\}$;
- $x \in (C_1 \mathcal{U} C_2)^{I(h,w)}$ iff there exists $v \in h$ such that $w < v$, $x \in C_2^{I(h,v)}$ and $x \in C_1^{I(h,u)}$ for all $u \in (w, v)$;
- $x \in (\mathsf{E}C)^{I(h,w)}$ iff there exists $h' \in \mathcal{H}$ such that $w \in h'$ and $x \in C^{I(h',w)}$.

The truth-relation $(\mathfrak{M}, h, w) \models \varphi$ (or $(h, w) \models \varphi$ for short) is now defined as in the first-order case. We give only the truth-conditions for atomic formulas:

- $(h, w) \models C_1 \sqsubseteq C_2$ iff $C_1^{I(h,w)} \subseteq C_2^{I(h,w)}$;
- $(h, w) \models a : C$ iff $a^{I(w)} \in C^{I(h,w)}$;
- $(h, w) \models aRb$ iff $a^{I(w)} R^{I(w)} b^{I(w)}$.

The other temporal description logics we are interested in are fragments of $\mathcal{DPCTL}^*$. For example, $\mathcal{DTL}$ is the linear time fragment of $\mathcal{DPCTL}^*$ obtained by omitting the path quantifiers. $\mathcal{DCTL}^*_F$ is the future fragment of $\mathcal{DPCTL}^*$ in which $\mathcal{U}$ is replaced with $\Box_F$. $\mathcal{DPCTL}$ is obtained by restricting the application of temporal operators and path quantifiers to concepts and formulas by allowing only the patterns $\mathsf{A}(E_1 \mathcal{U} E_2)$, $\mathsf{E}(E_1 \mathcal{U} E_2)$, $\mathsf{A}(E_1 \mathcal{S} E_2)$, and $\mathsf{E}(E_1 \mathcal{S} E_2)$.

All these logics can be regarded as fragments of first-order temporal logic having the object names $a, b, \ldots$ as their constants. Indeed, fix two different individual variables, say, x, y. The translation R_i^T of a role name R_i is the formula $R(x, y)$ with two free variables x, y. The translation C^T of a concept C is a formula with one free variable x defined by taking

$$
\begin{aligned}
A_i^T &= A_i(x) & (\neg C)^T &= \neg C^T \\
(C \sqcap D)^T &= C^T \wedge D^T & (C_1 \mathcal{U} C_2)^T &= C_1^T \mathcal{U} C_2^T \\
(\exists R.C)^T &= \exists y\, (R^T \wedge C^T\{y/x\}) & (\mathsf{A}C)^T &= \mathsf{A}C^T
\end{aligned}
$$

and similarly for $\mathcal{S}$ and E. The translation φ^T of a $\mathcal{DPCTL}^*$-formula φ is a sentence defined as follows:

$$
\begin{aligned}
(C \sqsubseteq D)^T &= \forall x\ (C^T \to D^T) & (a : C)^T &= C^T\{a/x\} \\
(aRb)^T &= R^T\{a/x, b/y\} & (\varphi \wedge \psi)^T &= \varphi^T \wedge \psi^T, \\
(\neg\varphi)^T &= \neg\varphi^T & (\mathsf{A}\varphi)^T &= \mathsf{A}\varphi^T
\end{aligned}
$$

and similarly for the other temporal operators. It is readily checked that a $\mathcal{DPCTL}^*$-formula φ is satisfiable iff its first-order translation φ^T is satisfiable.

3.2 Spatio-temporal logics

Our second example is a family of logics devised in [59, 62] for qualitative representation and reasoning about spatial regions moving in time. The logics are obtained by combining (propositional) temporal logics with the region connection calculus RCC-8.

Recall that RCC-8 contains eight binary relations between regions in topological spaces: DC (disconnection), EQ (equality), PO (partial overlap), EC (external connection), TTP (tangential proper part), NTTP (non-tangential proper part), and the inverses of the last two. Boolean operations are allowed on these relations. See, e.g., [47, 24, 25, 11, 58].

The intended semantics of RCC-8 is as follows. Region variables $X, Y, \dots$ are interpreted as regular closed sets in a topological space $\langle T, \mathbb{I} \rangle$. Here, $\mathbb{I}$ is the interior operator on the universe T, and $\mathbb{C}$ is the corresponding closure operator. A regular closed set $S \subseteq T$ is one satisfying $S = \mathbb{C}\mathbb{I}S$. Given such an interpretation (or assignment) of the region variables, we then put

$$
\begin{array}{ll}
\mathsf{DC}(X,Y) & \text{iff } \neg\exists x\ x \in X \cap Y,\\
\mathsf{EQ}(X,Y) & \text{iff } \forall x\ (x \in X \leftrightarrow x \in Y),\\
\mathsf{PO}(X,Y) & \text{iff } \exists x\ (x \in \mathbb{I}X \cap \mathbb{I}Y) \wedge \exists x\ (x \in \mathbb{I}X \cap -Y) \wedge \exists x\ (x \in -X \cap \mathbb{I}Y),\\
\mathsf{EC}(X,Y) & \text{iff } \exists x\ (x \in X \cap Y) \wedge \neg\exists x\ (x \in \mathbb{I}X \cap \mathbb{I}Y),\\
\mathsf{TPP}(X,Y) & \text{iff } \forall x\ (x \in -X \cup Y) \wedge \exists x\ (x \in X \cap -\mathbb{I}Y) \wedge \exists x\ (x \in -X \cap Y),\\
\mathsf{TPPi}(X,Y) & \text{iff } \mathsf{TPP}(Y,X),\\
\mathsf{NTPP}(X,Y) & \text{iff } \forall x\ (x \in -X \cup \mathbb{I}Y) \wedge \exists x\ (x \in -X \cap Y),\\
\mathsf{NTPPi}(X,Y) & \text{iff } \mathsf{NTPP}(Y,X).
\end{array}
$$

The maximal spatio-temporal language $\mathcal{SPCTL}^*$ ($\mathcal{S}$ for 'spatial') that we consider in this paper is defined as follows. *Region terms* are constructed from region variables by means of the boolean operators, $\mathcal{S}, \mathcal{U}$, and A. *Atomic formulas* are of the form $R(t_1, t_2)$, where t_1, t_2 are region terms and R an RCC-8 relation. Complex *formulas* are constructed from atomic ones using the booleans, $\mathcal{S}$, $\mathcal{U}$, and A. (E continues to abbreviate $\neg\mathsf{A}\neg$.) In this language, we can speak about the temporal development of relations between regions, as in

$$\mathsf{E}\Box_F \mathsf{TPP}(\mathit{Kosovo}, \mathit{Yugoslavia}) \wedge \mathsf{E} \bigcirc \mathsf{EC}(\mathit{Kosovo}, \mathit{Yugoslavia})$$

('it is possible that Kosovo will always be part of Yugoslavia and also possible that already in one year Kosovo will be a neighbor of Yugoslavia'). We can also speak about the dynamics of regions themselves. For example,

$$\Box_F\big(\mathsf{NTPP}(EU, \bigcirc EU) \vee \mathsf{TPP}(EU, \bigcirc EU)\big),$$

interpreted in discrete flows of time, means that the EU will never contract. As an example of a more complex statement, consider

$$\mathsf{DC}(\mathit{Russia}\ \mathcal{S}\ \mathit{Russian\_Empire}, \mathit{Russia}\ \mathcal{S}\ \mathit{Germany}),$$

which can be used to say that the part of Russia that has remained Russian since 1917 is not connected to the part of Germany (Königsberg) that became Russian after the Second World War.

Models for $\mathcal{SPCTL}^*$ are obtained from first-order temporal models by associating via I with every moment w not a first-order structure but a topological space together with an assignment $\mathfrak{a}_w$ for the region variables. Thus, $I(w)$ is a triple of the form $\langle T, \mathbb{I}, \mathfrak{a}_w \rangle$, where $\mathfrak{a}_w$ maps each region variable X to a regular closed subset of T. (Note that region variables are not rigid.)

Now define the *value* $t^{I(h,w)}$ of a region term t relative to a history h and a moment $w \in h$, as follows (we list only some clauses):

$$\begin{aligned}
X^{I(h,w)} &= \mathfrak{a}_w(X), \\
(\neg t)^{I(h,w)} &= \mathbb{CI}\,(T - t^{I(h,w)}), \\
(t_1 \sqcap t_2)^{I(h,w)} &= \mathbb{CI}\,(t_1^{I(h,w)} \cap t_2^{I(h,w)}), \\
(t_1 \mathcal{U} t_2)^{I(h,w)} &= \mathbb{CI}\,\{x : \exists v > w\, \big(v \in h\ \wedge x \in t_2^{I(h,v)} \wedge \forall u \in (w,v)\ x \in t_1^{I(h,u)}\big)\}, \\
(\mathsf{E}t)^{I(h,w)} &= \mathbb{CI}\,\{x : \exists h' \in \mathcal{H}\,(w \in h' \wedge x \in t^{I(h',w)})\}.
\end{aligned}$$

The right-hand side is always prefixed by $\mathbb{CI}$ because spatial regions are interpreted as regular closed sets.

The truth-relation $(h,w) \models \varphi$ is now defined in the obvious manner. For example, $(h,w) \models \mathsf{DC}(t_1,t_2)$ iff $\mathsf{DC}(t_1^{I(h,w)}, t_2^{I(h,w)})$ holds in $\langle T, \mathbb{I} \rangle$.

Example 1. Let φ be the conjunction of the following three $\mathcal{SPCTL}^*$-formulas:

$$\begin{aligned}
&\mathsf{P}(\textit{Kosovo}, \textit{Yugoslavia}) \\
&\mathsf{A}\Diamond_F \Box_F \mathsf{EC}(\textit{Kosovo}, \textit{Yugoslavia}) \\
&\mathsf{A}\Box_F \big(\mathsf{P}(\textit{Kosovo}, \textit{Yugoslavia}) \to \mathsf{E} \bigcirc \mathsf{P}(\textit{Kosovo}, \textit{Yugoslavia})\big)
\end{aligned}$$

The first formula means that at present Kosovo is part of Yugoslavia. The second says that in all possible histories, there'll be a time starting from which Kosovo will be externally connected to Yugoslavia. And the last formula claims that in all possible histories, it is always the case that if Kosovo is part of Yugoslavia then it is still possible that it will remain in Yugoslavia at least one more day. Clearly, φ is satisfiable in a bundled tree model but not in a full tree model.

Consider the following fragments of $\mathcal{SPCTL}^*$:

- $\mathcal{SCTL}^*_F$—the fragment without past operators and with $\Box_F$ instead of $\mathcal{U}$;
- $\mathcal{SCTL}^*_\bigcirc$—the fragment without past operators in which region terms are constructed using the booleans and $\bigcirc$ only;
- $\mathcal{STL}$—the fragment of $\mathcal{SPCTL}^*$ without path quantifiers;
- $\mathcal{STL}_\bigcirc$—the fragment of $\mathcal{STL}$ in which region terms are constructed using the boolean operators and $\bigcirc$.

It is not clear whether the full language $\mathcal{SPCTL}^*$ or its 'linear fragment' $\mathcal{STL}$ can be embedded into first-order temporal logics, because we have to deal with arbitrary topological spaces and infinite intersections and unions of sets. However, in some cases topological spaces induced by rather simple Kripke frames are enough. This is so for the languages $\mathcal{SCTL}^*_\bigcirc$ and $\mathcal{STL}_\bigcirc$. As for $\mathcal{SPCTL}^*$ and $\mathcal{STL}$, in many applications it is sufficient to consider satisfiability in models where every region can have only *finitely many* states—the so-called *FSA-models*. (FSA stands for the 'finite state assumption'.) In all these cases, the satisfiability problem for spatio-temporal formulas can be (polynomially) reduced to satisfiability of first-order temporal formulas with *only one individual variable*. This (rather non-trivial) reduction can be found in [62, 29].

4 Natural borders of decidability

The following theorems indicate some limits beyond which one cannot hope to find decidable fragments of first-order temporal logics.

Given a first-order temporal language $\mathcal{TL}$ and $\ell < \omega$, we denote by $\mathcal{TL}^\ell$ the *ℓ-variable fragment* of $\mathcal{TL}$ (i.e., every formula in $\mathcal{TL}^\ell$ contains at most ℓ distinct individual variables). And by $\mathcal{TL}^{mo}$ we denote the *monadic fragment* of $\mathcal{TL}$ (i.e., the set of formulas which contain only unary predicates and propositional variables). Both the two-variable and the monadic fragments of classical (non-temporal) first-order logic are known to be decidable and have the finite model property; see [13] and references therein. The computational behavior of the corresponding fragments of first-order temporal logics turns out to be quite different.

Theorem 1 **([35]).** *Let $\mathfrak{F}$ be either $\{\langle\mathbb{N},<\rangle\}$ or $\{\langle\mathbb{Z},<\rangle\}$. Then*

$$\mathcal{QTL}^2 \cap \mathcal{QTL}^{mo} \cap QTL(\mathfrak{F})$$

is not recursively enumerable.

Theorem 2 **([35]).** *Let $\mathcal{F}$ be one of the following classes of temporal frames: $\{\langle\mathbb{N},<\rangle\}$, $\{\langle\mathbb{Z},<\rangle\}$, the class of all strict linear orders. Then*

$$\mathcal{QTL}^2 \cap \mathcal{QTL}^{mo} \cap QTL_{fin}(\mathcal{F})$$

is not recursively enumerable.

Another well-behaved fragment of classical predicate logic is the *guarded fragment* of [5]. The corresponding fragment $\mathcal{TGF}$ of first-order temporal logic is the smallest set of $\mathcal{QPCTL}^*$-formulas such that:

- every atomic formula is in $\mathcal{TGF}$;
- if φ and ψ are in $\mathcal{TGF}$, then so are $\varphi \wedge \psi$, $\neg\varphi$, $\varphi\mathcal{S}\psi$, $\varphi\mathcal{U}\psi$, and $\mathsf{A}\varphi$;
- if $\overline{x}, \overline{y}$ are tuples of variables, $G(\overline{x}, \overline{y})$ is an atomic formula, $\varphi(\overline{x}, \overline{y}) \in \mathcal{TGF}$, and every free variable occurring in $\varphi(\overline{x}, \overline{y})$ occurs in $G(\overline{x}, \overline{y})$ as well, then $\forall\overline{y}\,(G(\overline{x}, \overline{y}) \to \varphi(\overline{x}, \overline{y}))$ is in $\mathcal{TGF}$.

The set $\mathcal{TGF}$ is called the *guarded fragment* of the first-order temporal language. Again in contrast to the case of classical predicate logic, we have the following:

Theorem 3 **([35]).** *Let $\mathfrak{F} \in \{\langle\mathbb{N},<\rangle, \langle\mathbb{Z},<\rangle\}$. Then $QTL(\mathfrak{F}) \cap \mathcal{QTL}^2 \cap \mathcal{TGF}$ is not recursively enumerable.*

The computational behavior of branching time logics is even worse:

Theorem 4 ([36]). *The one-variable fragments of $QCTL_F^*$ and $BQCTL_F^*$ are undecidable.*

Remark 3. A more general version of this result can be formulated in terms of products of propositional modal logics; see [29] and references therein. Namely, there is no decidable set L of formulas such that

$$\mathsf{K} \times \mathsf{CTL}_F^* \subseteq L \subseteq \mathsf{S5} \times \mathsf{CTL}_F^*.$$

The same holds for the propositional bundled case as well.

Problem 1. Which fragments of $BQPCTL^*$ and $QPCTL^*$ are recursively enumerable?

5 Decidable fragments

The negative results of the previous section can be 'explained' by the fact that all the undecidable fragments there are in a sense 'three-dimensional', which is often a cause of bad computational properties. The three-variable fragment of classical first-order logic is undecidable even without equality [41], and products of three propositional modal logics are usually undecidable [34]. In Theorems 1, 2, and 3, the linear time operator $\mathcal{U}$ can be applied to formulas with two free variables, and so we can quantify in three 'dimensions': one temporal and two domain. In Theorem 4 we also have quantification in three dimensions: temporal operators, path quantifiers and the domain quantification.

The following definition of [35, 61] suggests a way to avoid this kind of interaction.

Definition 1 (monodic formulas). Let $\mathcal{QPCTL}_1^*$ be the set of all $\mathcal{QPCTL}^*$-formulas φ such that any subformula of φ of the form $\psi_1\mathcal{U}\psi_2$, $\psi_1\mathcal{S}\psi_2$, or $\mathsf{A}\psi$ has at most one free variable. Such formulas φ will be called *monodic.* In other words, monodic formulas allow quantification into temporal contexts only with one free variable. $\mathcal{QPCTL}_1$, $\mathcal{QTL}_1$, and $\mathcal{TGF}_1$ denote the corresponding fragments of $\mathcal{QPCTL}$, $\mathcal{QTL}$, and $\mathcal{TGF}$ respectively.

Of course the monodic fragments of the logics under consideration are still undecidable, simply because they contain full first-order logic. However, the monodic fragments of $QTL(\langle\mathbb{N},<\rangle)$ and $QTL(\langle\mathbb{Z},<\rangle)$ become recursively enumerable: see [63] for axiomatizations.

Problem 2. Are the monodic fragments of other temporal logics considered in this paper (finitely) axiomatizable—in particular, the monodic fragments of $QCTL$ and $QPCTL^*$?

We now provide some general conditions under which fragments of the monodic temporal logics are decidable. To simplify presentation, we will assume that our languages contain no individual constants.

Fix a $\mathcal{QPCTL}_1^*$-formula φ. Denote by $sub_1\varphi$ the closure under (single) negation of the set of all subformulas of φ that contain at most one free variable. Let x be a variable not occurring in φ. Put

$$sub_x\varphi = \{\psi\{x/y\} : \psi(y) \in sub_1\varphi\}.$$

By a *type* for φ we mean any boolean-saturated subset t of $sub_x\varphi$: that is,

- $\psi \wedge \chi \in t$ iff $\psi \in t$ and $\chi \in t$, for every $\psi \wedge \chi \in sub_x\varphi$;
- $\neg\psi \in t$ iff $\psi \notin t$, for every $\neg\psi \in sub_x\varphi$.

For every $\mathcal{QPCTL}_1^*$-formula $\psi(x)$ starting with $\mathcal{U}$, $\mathcal{S}$, or A, and having one free variable x, we reserve a unary predicate $P_\psi(x)$. Similarly, for every sentence ψ of this form we fix a propositional variable p_ψ. These predicates and variables are called the *surrogates* of the original formulas. We denote by $\overline{\varphi}$ the result of replacing in φ all its subformulas starting with $\mathcal{U}$, $\mathcal{S}$, or A by their surrogates. Thus, $\overline{\varphi}$ is a formula of classical first-order logic. For a φ-type t, we write $\overline{t}$ for $\{\overline{\psi} : \psi \in t\}$; we identify $\overline{t}$ with its conjunction $\bigwedge \overline{t}$.

A set S of φ-types is called (*finitely*) *realizable* if the sentence

$$\alpha_S \quad = \quad \bigwedge_{t \in S} \exists x\, \overline{t}(x) \quad \wedge \quad \forall x \bigvee_{t \in S} \overline{t}(x)$$

is satisfiable (in a finite model).

Theorem 5 (criterion 1: linear case [35]). *Let $\mathcal{L}' \subseteq \mathcal{QTL}_1$.*

(i) *Let $\mathcal{F}$ be any of the following classes of flows of time:*

- $\{\langle \mathbb{N}, < \rangle\}$,
- $\{\langle \mathbb{Z}, < \rangle\}$,
- $\{\langle \mathbb{Q}, < \rangle\}$,
- *the class of all finite strict linear orders,*
- *any first-order-definable class of strict linear orders.*

Suppose there is an algorithm that decides for any $\mathcal{L}'$-formula φ whether an arbitrarily-given set of φ-types is realizable. Then the satisfiability problem for $\mathcal{L}'$-sentences in models based on flows of time in $\mathcal{F}$ is decidable.

(ii) *Let $\mathcal{F}^+$ range over the classes of flows of time above and $\{\langle \mathbb{R}, < \rangle\}$, and suppose that finite realizability is decidable for sets of types of formulas in $\mathcal{L}'$. Then the satisfiability problem for $\mathcal{L}'$-sentences in models based on flows of time in $\mathcal{F}$ and having finite domains is decidable.*

The proof of part (i) of the theorem uses *quasimodels.* A quasimodel of φ over a flow of time $\mathfrak{F} = \langle W, < \rangle$ consists of an assignment of a realizable set of types S_w to each $w \in W$, the sequence $\langle S_w : w \in W \rangle$ having certain specified properties. The existence of a quasimodel for φ can be expressed by a sentence σ_φ of monadic second-order logic, which can be effectively constructed from φ so long as it is decidable whether an arbitrary set of φ-types is realizable. It is shown that φ is satisfiable in a model with flow of time $\mathfrak{F}$ iff there exists a quasimodel for φ over $\mathfrak{F}$. Thus, φ has a model with flow of time $\mathfrak{F}$ iff $\mathfrak{F} \models \sigma_\varphi$. If $\mathfrak{F}$ is $\{\langle \mathbb{N}, <\rangle\}$, $\{\langle \mathbb{Z}, <\rangle\}$, or $\{\langle \mathbb{Q}, <\rangle\}$, this last statement is decidable by results of [15, 46]. The other cases can now be obtained by reduction. Part (ii) is proved differently, since the full monadic second-order theory of $\langle \mathbb{R}, <\rangle$ is undecidable. The proof adapts the argument of the second part of [17]: see [35] for details.

As a consequence of Theorem 5 we obtain, for example, the following results, where $\mathcal{F}$ and $\mathcal{F}^+$ have the same meaning as above.

Theorem 6. *The following fragments are decidable:*

$$
\begin{array}{ll}
QTL(\mathcal{F}) \cap \mathcal{QTL}^1, & QTL_{fin}(\mathcal{F}^+) \cap \mathcal{QTL}^1, \\
QTL(\mathcal{F}) \cap \mathcal{QTL}_1^2, & QTL_{fin}(\mathcal{F}^+) \cap \mathcal{QL}_1^2, \\
QTL(\mathcal{F}) \cap \mathcal{QTL}_1^{mo}, & QTL_{fin}(\mathcal{F}^+) \cap \mathcal{QTL}_1^{mo}, \\
QTL(\mathcal{F}) \cap \mathcal{TGF}_1, & QTL_{fin}(\mathcal{F}^+) \cap \mathcal{TGF}_1.
\end{array}
$$

The corresponding loosely guarded and fluted fragments are decidable as well (for definitions see [61]).

Problem 3. Does Theorem 6 hold for $QTL(\langle \mathbb{R}, <\rangle)$?

Problem 4. What is the computational complexity of the decision problem for the above fragments as well as other decidable fragments mentioned in this paper?

It follows from the proof of Theorem 5 in [35] for the flow of time $\langle \mathbb{N}, <\rangle$ that $QTL(\langle \mathbb{N}, <\rangle) \cap \mathcal{L}'$ is in EXPSPACE whenever the problem of whether a set of φ-types, for an $\mathcal{L}'$-formula φ, is realizable is decidable in EXPSPACE (in the length of φ). Conversely, it is proved in [29] that $QTL(\langle \mathbb{N}, <\rangle) \cap \mathcal{QTL}^1$ is EXPSPACE-hard. It follows, for example, that the satisfiability problem for $\mathcal{QTL}_1^{mo}$-formulas in models based on $\langle \mathbb{N}, <\rangle$ is EXPSPACE-complete.

According to Theorem 4, the results above do not generalize to $\mathcal{QPCTL}^*$, since already the one-variable fragment of $\mathcal{QCTL}_F^*$ is undecidable. However, as soon as we restrict ourselves to $\mathcal{QPCTL}$, we obtain the following:

Theorem 7 (criterion 2: branching case [36]). *Let $\mathcal{L}' \subseteq \mathcal{QPCTL}_1$ and suppose that there is an algorithm that decides for any $\mathcal{L}'$-formula φ whether an arbitrarily-given set of φ-types is realizable. Then satisfiability of $\mathcal{L}'$-formulas is decidable for both bundled and full tree models.*

The proof is again via quasimodels and embedding into monadic second-order logic.

As a consequence we obtain

Theorem 8. *The fragments*

$$QPCTL \cap \mathcal{QPCTL}_1^2, \quad QPCTL \cap \mathcal{QPCTL}_1^{mo}, \quad QPCTL \cap \mathcal{TGF}_1,$$

as well as their bundled versions are decidable.

Another way to obtain decidable fragments in the branching time case is to restrict further the formulas that can have free variables. Denote by $\mathcal{QCTL}^*_{\bigcirc}$ the fragment of $\mathcal{QCTL}^*_1$ in which only $\bigcirc$ may be applied to formulas with one free variable (the other operators $\mathcal{U}$, E and A may be applied only to sentences). Then we have:

Theorem 9 (criterion 3: branching case [36]). *Let $\mathcal{L}' \subseteq \mathcal{QCTL}^*_{\bigcirc}$ and suppose that there is an algorithm that decides for any $\mathcal{L}'$-formula φ whether an arbitrarily-given set of φ-types is realizable. Then satisfiability of $\mathcal{L}'$-formulas in bundled trees models is decidable.*

Here the proof uses quasimodels and a mosaic technique.

Therefore, we also obtain

Theorem 10. *The fragments*

$$BQCTL^* \cap (\mathcal{QCTL}^*_{\bigcirc})^2, \quad BQCTL^* \cap (\mathcal{QCTL}^*_{\bigcirc})^{mo}, \quad BQCTL^* \cap \mathcal{TGF}_{\bigcirc}$$

are decidable.

Problem 5. Does Theorem 10 hold for the unbundled case?

Problem 6. Are there any interesting languages between $\mathcal{QCTL}^*_{\bigcirc}$ and $\mathcal{QCTL}^*$ for which Theorem 10 holds? In particular, does it hold for the fragment of those $\mathcal{QCTL}^*$-formulas in which $\mathcal{U}$ can be applied to formulas with one free variable but A only to closed formulas? Does it hold for the fragment of those $\mathcal{QCTL}^*$-formulas in which $\bigcirc$ and E can be applied to formulas with one free variable but $\mathcal{U}$ only to closed formulas?

6 Adding equality

So far we have considered first-order temporal languages *without equality and function symbols*. A natural question is whether our decidability results concerning the class of monodic formulas can be generalized to the languages with these ingredients.

It is not hard to see (cf. [63]) that adding function symbols quickly destroys the nice properties of monodic formulas. For example, the set of one-variable formulas with one function symbol that are valid in models based on $\langle \mathbb{N}, < \rangle$ is not recursively enumerable.

Let us now consider the language $\mathcal{QTL}$ extended with equality (but not function symbols). This language will be denoted by $\mathcal{QTL}^=$. The following negative result can be proved by reduction of the validity problem for first-order formulas in finite structures.

Theorem 11 (**[61]**)**.** *The set of monodic $\mathcal{QTL}^{=}$-formulas that are valid in all temporal models based on $\langle \mathbb{N}, < \rangle$ is not recursively enumerable, and so not recursively axiomatizable.*

Recently, this result was generalized to fragments that are decidable without equality:

Theorem 12 (**[22]**)**.** *The set of two-variable monadic formulas in $\mathcal{QTL}_1^{=}$ that are valid in models based on $\langle \mathbb{N}, < \rangle$ is not recursively enumerable.*

The guarded fragment (and extensions like the loosely guarded, packed, and clique-guarded [33] fragments) are more robust under addition of equality:

Theorem 13 (**[37]**)**.** *$QTL^{=}(\mathcal{F}) \cap \mathcal{QTL}_1 \cap \mathcal{TGF}$ and $QTL^{=}_{fin}(\mathcal{F}^{+}) \cap \mathcal{QTL}_1 \cap \mathcal{TGF}$ are decidable, where $\mathcal{F}, \mathcal{F}^{+}$ are as in theorem 5.*

The reason for this robustness is that the models of guarded sentences are closed under disjoint unions. The same property was used in [60] to show that various temporalized description logics with number restrictions and nominals are decidable.

7 Applications

The results obtained for monodic fragments of first-order temporal logics can be applied to both temporal description logics and spatio-temporal logics.

7.1 Temporal description logics

Observe that the translation φ^T (defined in Section 5) of any $\mathcal{DTL}$-formula φ lies in the two-variable fragment of the monodic fragment of $\mathcal{QTL}$.[2] Hence, we obtain the following results; cf. [60].

Theorem 14. *Suppose that $\mathcal{F}$ is a class of strict linear orders. If satisfiability of $\mathcal{QTL}_1 \cap \mathcal{QTL}^2$-formulas in models based on flows of time in $\mathcal{F}$ is decidable, then satisfiability of $\mathcal{DTL}$-formulas in $\mathcal{F}$ is decidable as well.*

This decidability result extends to temporalized description logics based on more expressive description logics than $\mathcal{ALC}$. For example, one can take instead of $\mathcal{ALC}$ the description logic $\mathcal{CI}$ corresponding to propositional dynamic logic with converse (see [60]) or the description logic $\mathcal{DLR}$ of [19] which contains n-ary roles. The resulting temporalized description logics are suitable for temporal conceptual modelling [8].

The situation is quite different in the branching case. One can show (see [29] for details) that the one-variable fragment of $\mathcal{QCTL}^*_F$ is embeddable into $\mathcal{DCTL}^*_F$. Hence we obtain from Theorem 4:

[2] Note that we would not obtain monodic formulas if temporal operators were applied to roles. This is another explanation of the bad behavior of temporal description logics with temporalized roles.

Theorem 15. *Satisfiability of $\mathcal{DCTL}^*_F$-formulas is undecidable both in bundled and full ω-trees.*

To obtain positive results in the branching case, we can employ Theorem 8. The translation φ^T of any $\mathcal{DPCTL}$-formula φ belongs to $\mathcal{QPCTL}_1 \cap \mathcal{QPCTL}^2$. So we have:

Theorem 16. *Satisfiability of $\mathcal{DPCTL}$-formulas is decidable both in bundled and full ω-trees.*

7.2 Spatio-temporal logics

We now apply the results about first-order temporal logics to spatio-temporal logics. Consider first the linear case.

Theorem 17 ([59]). *Suppose satisfiability of the one-variable fragment of $\mathcal{QTL}$ in the class $\mathcal{F}$ of linear flows of time is decidable. Then satisfiability of $\mathcal{STL}_{\bigcirc}$-formulas in $\mathcal{F}$ is decidable.*

Problem 7. Is the satisfiability problem for arbitrary $\mathcal{STL}$-formulas in models based on $\langle \mathbb{N}, < \rangle$ and other flows of time decidable?

The difficulty here is that the application of temporal operators different from $\bigcirc$ corresponds on the semantic side to possibly infinite intersections and unions of sets. These are not respected by the embeddings into first-order temporal logics. However, as far as we are satisfied with FSA-models (in which each region may have only finitely many states), we can reduce the satisfiability problem to satisfiablity of one-variable first-order formulas in models with *finite* domains. We then obtain:

Theorem 18 ([62]). *Suppose that satisfiability of the one-variable fragment of $\mathcal{QTL}$ in models based on flows of time from a class $\mathcal{F}$ and having finite domains is decidable. Then satisfiability of $\mathcal{STL}$-formulas in FSA-models based on flows of time in $\mathcal{F}$ is decidable as well. This applies, in particilar, to all classes $\mathcal{F}^+$ mentioned in Theorem 5 (ii).*

The branching case is again different because we can encode the one-variable fragment of $\mathcal{QCTL}^*_F$ in $\mathcal{SCTL}^*_F$, even if we confine ourselves to discrete topological spaces. Thus we obtain from Theorem 4:

Theorem 19 ([36]). *Satisfiability of $\mathcal{SCTL}^*_F$-formulas is undecidable for both bundled and full ω-trees, even for models based on discrete topological spaces.*

$\mathcal{SCTL}^*_{\bigcirc}$ can be embedded into the one-variable fragment of $\mathcal{QCTL}^*_{\bigcirc}$. So we obtain from Theorem 10:

Theorem 20 ([36]). *Satisfiability of $\mathcal{SCTL}^*_{\bigcirc}$-formulas in bundled ω-trees is decidable.*

8 Practical reasoning algorithms

Two kinds of 'practical' algorithms have been proposed for logics related to monodic fragments of first-order temporal logics. [52, 40] develop tableau-based decision procedures for temporal description logics which combine standard tableaux for description logics with Wolper's [55] tableau for propositional temporal logic. An interesting aspect of those tableau calculi is that they are based on the same idea as the model-theoretic decidability proofs. Instead of constructing models directly, they construct quasimodels in which the states are locally saturated sets of (partial) types. A tableau calculus for temporal description logics with *expanding domains* is implemented in [32]. Those algorithms are easily modified to obtain an algorithm deciding the future-fragment of $QTL(\langle \mathbb{N}, < \rangle) \cap \mathcal{QTL}^1$.

The second approach employs the resolution method. [23] provides a clausal temporal resolution calculus which is complete for those formulas of the future fragment of $QTL(\langle \mathbb{N}, < \rangle) \cap \mathcal{QTL}_1$ in which only $\bigcirc$ is applied to open formulas (other temporal operators may be applied only to sentences).

9 Expressive completeness

The temporal language $\mathcal{QTL}$ provides only 'implicit' access to time. Quantification over points in time in the sense of first-order logic is not permitted, and the only means of expressing temporal properties is by the operators Since and Until. A common alternative is to reason about time explicitly, using first-order logic. Following this approach in the propositional case yields monadic first-order logic interpreted in strict linear orders, while in the predicate case it leads to a two-sorted first-order language, called '$\mathcal{TS}$' in what follows, one sort of which refers to points in time and the other to the first-order domain (see, e.g., [2, 3, 20, 21]).

The alphabet of $\mathcal{TS}$ consists of:

- an infinite set of individual variables $x_0, x_1, \dots$, and a set of constants $c_0, c_1, \dots$ of domain sort,
- an infinite set of individual variables $t_0, t_1, \dots$ of temporal sort,
- the binary predicate symbol $<$ of sort 'temporal $\times$ temporal',
- predicate symbols $P_0, P_1, \dots$ of sort 'temporal $\times$ domain^n', $n < \omega$.

Formulas of $\mathcal{TS}$ are defined inductively:

- $t_i < t_j$ is an (atomic) formula, for temporal variables t_i, t_j;
- $P(t, y_1, \dots, y_n)$ is an (atomic) formula, for a predicate symbol P of sort temporal $\times$ domain^n, a temporal variable t, and domain variables or constants $y_1, \dots, y_n$;
- if φ and ψ are formulas and v is a (temporal or domain) variable, then $\neg\varphi$, $\varphi \wedge \psi$, and $\forall v \varphi$ are formulas.

$\mathcal{TS}$ is interpreted in standard first-order temporal models $\mathfrak{M} = \langle \mathfrak{F}, D, I \rangle$, where $\mathfrak{F} = \langle W, < \rangle$ is a flow of time (i.e., a strict linear order), D is a non-empty set, the *domain* of $\mathfrak{M}$, and I is a function associating with every moment of time $w \in W$ a first-order structure

$$I(w) = \left\langle D, P_0^{I(w)}, \ldots, c_0^{I(w)}, \ldots \right\rangle,$$

in which, for each i, $P_i^{I(w)}$ is a predicate on D of arity n whenever P_i is of arity $n+1$, and $c_i^{I(w)} \in D$. We require again that $c_i^{I(w)} = c_i^{I(w')}$ for all $w, w' \in W$.

An *assignment* in $\mathfrak{M}$ is a function $\mathfrak{a} = \mathfrak{a}_1 \cup \mathfrak{a}_2$ such that $\mathfrak{a}_1$ associates with every temporal variable t a moment of time $\mathfrak{a}_1(t) \in W$ and $\mathfrak{a}_2$ associates with every domain variable x an element $\mathfrak{a}_2(x)$ of D. We extend $\mathfrak{a}$ canonically to constants as before.

The truth relation $\mathfrak{M} \models^{\mathfrak{a}} \varphi$ is defined inductively as follows:

- $\mathfrak{M} \models^{\mathfrak{a}} t_i < t_j$ iff $\mathfrak{a}_1(t_i) < \mathfrak{a}_1(t_j)$;
- $\mathfrak{M} \models^{\mathfrak{a}} P(t, y_1, \ldots, y_n)$ iff $\langle \mathfrak{a}_2(y_1), \ldots, \mathfrak{a}_2(y_n) \rangle \in P^{I(\mathfrak{a}_1(t))}$, for any constants or domain variables $y_1, \ldots, y_n$;
- the standard clauses for the booleans;
- $\mathfrak{M} \models^{\mathfrak{a}} \forall v \varphi$ iff $\mathfrak{M} \models^{\mathfrak{b}} \varphi$ for every assignment $\mathfrak{b}$ that may differ from $\mathfrak{a}$ only on v.

It should be clear that the temporal operators $\mathcal{U}$ and $\mathcal{S}$ of $\mathcal{QTL}$ are expressible in $\mathcal{TS}$. Formally, suppose that each n-ary predicate symbol Q_i of $\mathcal{QTL}$ is associated with the $(n+1)$-ary predicate symbol P_i of $\mathcal{TS}$. Also fix a temporal variable t. Define a translation $\dagger$ from $\mathcal{QTL}$ into $\mathcal{TS}$ by taking

$$\begin{aligned}
Q_i(x_1, \ldots, x_n)^\dagger &= P_i(t, x_1, \ldots, x_n), \\
(\varphi \wedge \psi)^\dagger &= \varphi^\dagger \wedge \psi^\dagger, \\
(\neg \varphi)^\dagger &= \neg(\varphi^\dagger), \\
(\forall x \varphi)^\dagger &= \forall x (\varphi^\dagger), \\
(\psi \mathcal{U} \varphi)^\dagger &= \exists t' (t < t' \wedge \varphi^\dagger\{t'/t\} \wedge \forall t'' (t < t'' < t' \to \psi^\dagger\{t''/t\})), \\
(\psi \mathcal{S} \varphi)^\dagger &= \exists t' (t' < t \wedge \varphi^\dagger\{t'/t\} \wedge \forall t'' (t' < t'' < t \to \psi^\dagger\{t''/t\})),
\end{aligned}$$

where t' and t'' are new temporal variables.

The meaning of the translation $\dagger$ can be explained as follows. Suppose that $\mathfrak{M} = \langle \mathfrak{F}, D, I \rangle$ is a $\mathcal{TS}$-model and that $\mathfrak{a} = \mathfrak{a}_1 \cup \mathfrak{a}_2$ is an assignment in $\mathfrak{M}$. Let $\mathfrak{N} = \langle \mathfrak{F}, D, J \rangle$ be a $\mathcal{QTL}$-model, and $\mathfrak{b}$ an assignment in $\mathfrak{N}$. We say that $\langle \mathfrak{M}, \mathfrak{a} \rangle$ and $\langle \mathfrak{N}, \mathfrak{b} \rangle$ are *equivalent* if $P_i^{I(w)} = Q_i^{J(w)}$ for all w, i, and $\mathfrak{a}_2 = \mathfrak{b}$.

Lemma 1. *Suppose that $\langle \mathfrak{M}, \mathfrak{a} \rangle$ and $\langle \mathfrak{N}, \mathfrak{b} \rangle$ are equivalent. Then for every $\mathcal{QTL}$-formula φ, we have $\langle \mathfrak{N}, \mathfrak{a}(t) \rangle \models^{\mathfrak{b}} \varphi$ iff $\mathfrak{M} \models^{\mathfrak{a}} \varphi^\dagger$.*

The proof is an easy induction on φ.

On the other hand, there are $\mathcal{TS}$-formulas that are not expressible in $\mathcal{QTL}$ over any interesting class of flows of time. For example, the $\mathcal{TS}$-sentence

$$\exists t_1 \exists t_2 \big(t_1 < t_2 \wedge \forall x (P(t_1, x) \leftrightarrow P(t_2, x))\big)$$

cannot be expressed in $\mathcal{QTL}$ over the flow of time $\langle \mathbb{Q}, < \rangle$, nor over the class of finite linear flows [39, 2, 3]. Recall that in the propositional case, both languages are known to have the same expressive power over most classes of flows of time—i.e., the temporal propositional language is *expressively complete*, see [38, 28]. It turns out, however, that $\mathcal{QTL}$ and $\mathcal{QTL}_1$ are expressively complete for some natural fragments of $\mathcal{TS}$.

Definition 2. *Let $\mathcal{TS}_{1t}$ (respectively, $\mathcal{TS}_{1x}$) consist of all $\mathcal{TS}$-formulas φ with no subformulas of the form $\forall x \psi$ ($\forall t \psi$) where ψ has more than one free temporal (respectively, domain) variable. Let $\mathcal{TS}_1 = \mathcal{TS}_{1t} \cap \mathcal{TS}_{1x}$.*

Note that for every $\mathcal{QTL}$-formula φ, we have $\varphi^\dagger \in \mathcal{TS}_{1t}$, and for every $\varphi \in \mathcal{QTL}_1$ we have $\varphi^\dagger \in \mathcal{TS}_1$.

Let $\mathcal{F}$ be a class of flows of time, $\mathcal{L}' \subseteq \mathcal{QTL}$, and $\mathcal{L}'' \subseteq \mathcal{TS}$. We say that $\mathcal{L}'$ is *expressively complete* for $\mathcal{L}''$ over $\mathcal{F}$ if for every $\varphi \in \mathcal{L}''$ with at most one free temporal variable, there exists a formula $\widehat{\varphi} \in \mathcal{L}'$ such that $(\widehat{\varphi})^\dagger$ and φ are equivalent in all models based on flows of time in $\mathcal{F}$.

Theorem 21 ([35]). *Let $\mathcal{F}$ be the class of all dedekind-complete linear flows of time ($\mathcal{F}$ contains, for example, $\langle \mathbb{N}, < \rangle$, $\langle \mathbb{Z}, < \rangle$, $\langle \mathbb{R}, < \rangle$, and all finite linear orders). Then*

1. *$\mathcal{QTL}$ is expressively complete for $\mathcal{TS}_{1t}$ over $\mathcal{F}$,*
2. *$\mathcal{QTL}_1$ is expressively complete for $\mathcal{TS}_1$ over $\mathcal{F}$.*

The proof uses Kamp's theorem ([38]; see also [28, chapters 9–12]) that the propositional temporal logic with $\mathcal{S}$ and $\mathcal{U}$ is expressively complete for monadic first-order logic over $\mathcal{F}$. The required $\widehat{\varphi}$ can be constructed effectively from φ.

For a class $\mathcal{H}$ of flows of time, denote by $TS(\mathcal{H})$ the set of all $\mathcal{TS}$-sentences that are true in all models based on frames in $\mathcal{H}$, and by $TS_{fin}(\mathcal{H})$ the set of $\mathcal{TS}$-sentences true in all models based on frames in $\mathcal{H}$ and having finite domains. Given a set $\mathcal{QTL}' \subseteq \mathcal{QTL}_1$, let

$$\mathcal{TS}' = \{\varphi \in \mathcal{TS}_1 : \widehat{\varphi} \in \mathcal{QTL}'\},$$

where $\widehat{\varphi}$ is as above. Since $\widehat{\varphi}$ can be constructed effectively from φ, by Lemma 1 and Theorem 21 we obtain the following:

Theorem 22. *Suppose that every $\mathfrak{F} \in \mathcal{H}$ be dedekind-complete, and let $\mathcal{QTL}' \subseteq \mathcal{QTL}_1$. If the fragment $QTL(\mathcal{H}) \cap \mathcal{QTL}'$ is decidable, then the fragment $TS(\mathcal{H}) \cap \mathcal{TS}'$ is decidable. If the fragment $QTL_{fin}(\mathcal{H}) \cap \mathcal{QTL}'$ is decidable, then the fragment $TS_{fin}(\mathcal{H}) \cap \mathcal{TS}'$ is decidable.*

This can be combined with Theorem 6 to obtain decidable fragments of $\mathcal{TS}$.

Problem 8. Extend the above results to cover branching time.

See [28] for information about expressive completeness for propositional temporal logics over branching time.

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