A Two-Player Game of Life

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Abstract

We present a new extension of Conway's game of life for two players, which we call *p2life*. P2life allows one of two types of token, black or white, to inhabit a cell, and adds competitive elements into the birth and survival rules of the original game. We solve the mean-field equation for p2life and determine by simulation that the asymptotic density of p2life approaches 0.0362.

Keywords: Two-player game of life; Cellular automata; Mean-field theory; Asymptotic density.

1 Introduction

Conway's game of life [1] is the best known two-dimensional cellular automaton [2], due to the complex behaviour it generates from a simple set of rules. The game of life, introduced to the world at large by Martin Gardner in 1970 [3], has provided throughout the years challenging problems to many enthusiasts. A large number of these are documented in Stephen Silver's comprehensive life lexicon [4]. Detailed accounts of the game of life have been given by Poundstone in [5], which uses the game of life as an illustration of complexity, and by Sigmund in [6], which uses the game of life in the context of artificial life and the ability of a cellular automaton to self-replicate. An interesting extension of the game of life to three dimensions was suggested by Bays [7].

The game of life is played on a square lattice with interactions to nearest and to next-nearest neighbours, where each cell can be either empty or occupied by a token and is surrounded by eight neighbouring cells. The evolution of the game is governed by the following simple rules. If a cell is empty it gives *birth* to a token if exactly three of its neighbours are occupied, and if it is occupied it *survives* if either two or three of its neighbours are occupied. In all other cases either the cell remains empty or it *dies*, i.e. it becomes empty. The game evolves by repeated applications of these rules to produce further configurations. The single player of the game decides what the initial configuration of the lattice will be and then watches the game evolve. One of the questions that researchers have investigated is the asymptotic behaviour of this evolutionary process. It transpires that when the initial configuration is random and its density is high enough, then the game eventually stabilises to a density of about 0.0287 [8, 9]; see also [10].

One aspect that is missing from Conway's game of life is the competitiveness element of two-player games, as Gardner noted "Attempts have also been made to invent competitive games based on "Life", for two or more players, but so far without memorable results." [11, p. 255]. This is the challenge that we take up in this paper.

An interesting version of the game of life for two players is known as *black and white* or *immigration* [4], where cells are either white or black and when a birth occurs the colour of the token is decided according to the majority of neighbouring cells. Although this variation is interesting in its own right, survival does not involve the two colours and remains non-competitive as in the single player version. We propose a new two-player version of the game of life where both birth and survival are competitive, and provide a preliminary analysis of its behaviour.

The rest of the paper is organised as follows. In Section 2 we present the rule set for our two-player version of the game of life and in Section 3 we provide a mean-field analysis of the game. In Section 4 we present results of simulations to ascertain the asymptotic density of the game and, finally, in Section 5 we give our concluding remarks and provide a web link to an implementation of the game.

2 Rules of the Game

In the two-player variation of the game of life, which we call *p2life*, the players, white and black, are competing for space. Conway's game of life is considered to be "interesting" since its simple set of rules lead to complex and unpredictable behaviour. (The notion that from simple rules complex behaviour can emerge, which may help us understand the diversity of natural phenomena, is discussed in great detail by Wolfram in [12] but can already be learned from the 130 year old thesis of van der Waals on liquid-vapour equilibria.) P2life maintains the "interesting" behaviour of the game of life by preserving the essence of Conway's game and adding to its rules a competitive element to decide who will give birth and who will survive.

The rules of p2life, from white's point of view (the rules from black's point of view are symmetric), are as follows:

Birth. If a cell is empty, then we consider two cases:

1) The cell has exactly three white neighbours and the number of black neighbours is different from three. In this case a white token is born in the cell.
2) The cell has exactly three white and three black neighbours. In this case an unbiased coin determines whether a white or black token is born in the cell.

Survival. If a cell is occupied by a white token, then we consider two cases:

1) If the difference between the number of white and black neighbours is two or three, then the white token survives.
2) If the difference between the number of white and black neighbours is one and the number of white neighbours is at least two, then the white token survives.

It is clear that if there is only one colour on the lattice then p2life reduces to the standard one-player version of the game. We note that having non-symmetric rules for white and black would allow us to investigate how different behaviour sets interact, but finding such a set of rules which would be "interesting" is an open problem. During the process of deciding the rule set for p2life we considered several variations, which we now briefly discuss:

1) In the first part of the birth rule, insisting that the number of black tokens should be less than three, which is similar to birth in the black and white variant. This decreases the birth rate but still seems to be "interesting".

2) Omitting the second part of the birth rule, i.e. the random choice when there are exactly three white and three black neighbours. Again, this decreases the birth rate but still seems to be "interesting".

3) Omitting the second part of the survival rule, i.e. survival when the difference between the number of white and black tokens is one and the number of white tokens is at least two. This decreases the survival rate and the conflict between the two players but, once again seems "interesting"

4) In the second part of the survival rule omitting the side condition that the number of white neighbours must be at least two, or in the first part of the survival rule omitting the condition that the difference between white and black is at most three. Due to the increased survival rates, these modifications lead to steady growth with spatial boundaries between the two players. It would be interesting to compare these rules to Schelling's models of segregation [13] or the Ising model [14].

We illustrate two configurations which lead to interesting confrontations between white and black. The configuration shown on the left-hand side of Table 1 leads to a black block and two white gliders as shown on the right-hand side of the table. In the one-player game of life this initial configuration annihilates itself into empty space. On the other hand, the configuration shown on the left-hand side of Table 2 leads to two black blocks as shown on the right-hand side of the table. Thus black wins from this position. In the one-player game of life this initial configuration leads to six blinkers.

●	●
●	●
○	○
○	○
○	○
○	○
○	○

$\Longrightarrow$

	○				●	●				○	
○	○				●	●				○	○
○		○							○		○

Table 1: An interesting configuration

●	●	●	●	●	●
		○	○		
		○	○		

$\Longrightarrow$

●	●			●	●
●	●			●	●

Table 2: Another interesting configuration

3 Mean-Field Theory for P2life

To compute the initial configuration given a specified initial density of p, we use the following procedure for each cell in the lattice. Firstly we determine whether it is occupied or empty according to the given probability p and then, if it should be occupied we toss an unbiased coin to decide whether it will be occupied by a white or black token. In a similar fashion to the game of life [8], we now use mean-field theory to determine the density of the tokens after applying the rules to the initial configuration.

It can be verified by the rules of p2life that the density p', after a single application of the rules to a square lattice having initial density p, is given the mean-field equation,

$$p' = 21\,p^3 - 63\,p^4 + \frac{105}{2}\,p^5 + 35\,p^6 - \frac{1505}{16}\,p^7 + \frac{1057}{16}\,p^8 - \frac{553}{32}\,p^9. \tag{1}$$

From (1) we can compute the maximum density of p', which is 0.3895 at an initial density of $p = 0.6206$. This can also be seen from the plot of the mean-field equation shown in Figure 1. As we would expect, simulations show a very close match with this plot, although obviously due to correlations we cannot use the mean-field equation to predict the long-term behaviour of the game. An interesting point to note about the plot is that when $p = 1$, $p' = 0.2188$, which is different from the original game of life (and the black and white variant) where $p' = 0$. The reason is that although dense regions of a single colour die off immediately, mixed spaces between the two colours allow for survival of tokens. We also note that from Figure 1 we can verify that the only fixed-point of the mean-field equation for p2life is zero. To improve the prediction power for repeated applications of the rules we could extend the mean-field analysis using the local structure theory of Gutowitz and Victor [15].

4 Asymptotic Density for P2life

We have investigated the properties of p2life with particular emphasis on the estimation of its asymptotic density via simulations using MATLAB. This approach offers flexibility and reasonable performance (0.7 million updates per second) for a particularly computationally intensive task. We have performed simulations on both periodic (toroidal) and cutoff boundary conditions on square lattices of sizes from 10×10 to 1000×1000. In particular, we have investigated the dependence of the asymptotic density p_∞ on the initial density p of a random, uniformly and independently distributed, initial configuration. Each configuration is iterated until the game configuration reaches a stable or oscillatory state. In general, it is straightforward to detect when such conditions occur, with the notable exception of the case of periodic boundary conditions when in the final configuration there exists a glider which travels around the torus without colliding with other tokens in occupied cells.

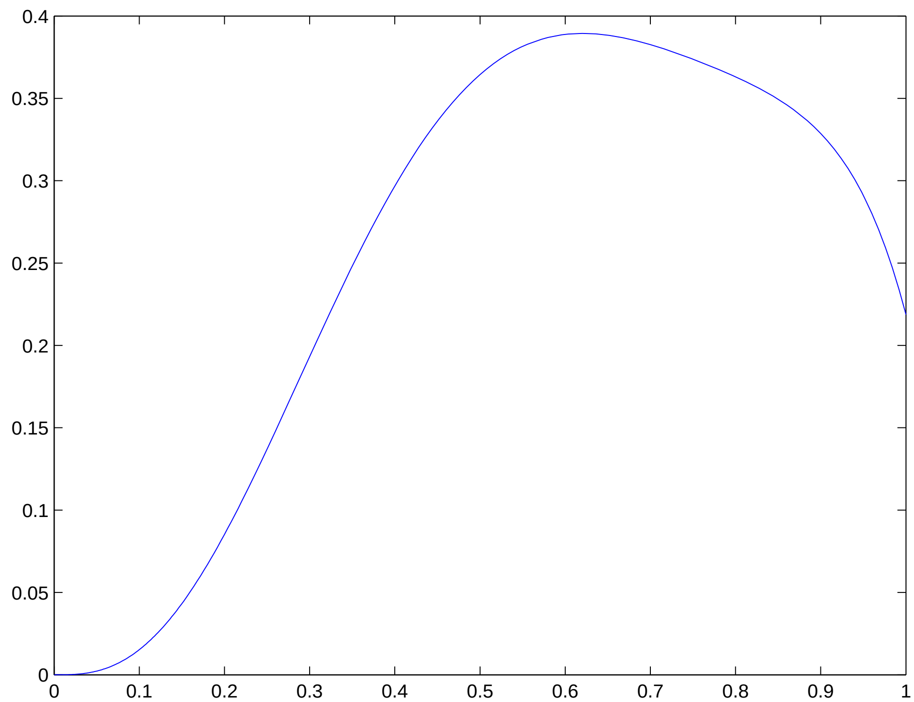

Figure 1: Plot of the mean-field density p' against the initial density p

The results for a square lattice of size 100×100 are shown in Figure 2. Each experimentally computed point plotted in Figure 2 is the average over one hundred independently selected initial configurations. It is immediately evident that in direct contrast to the one-player version of the game of life, the asymptotic density of p2life is non-zero for non-zero initial density. In the one-player version of the game, if the initial density increases above approximately 0.85 the asymptotic density is zero due to the annihilation of all tokens during the first iteration due to overcrowding. In fact, for $p = 1$ we estimate that p2life has asymptotic density $p_\infty = 0.0362$. This is due to the fact that in p2life the survival rules allow members of both token populations to carry over to the next iteration with $p' = 0.2188$, which is consistent with the value obtained via the mean-field approach.

We make several other interesting observations on the asymptotic behaviour of p2life. Firstly, we have estimated that the maximum asymptotic density of $p_\infty = 0.0362$ is reached at about $p = 0.3$ and remains fairly constant until $p = 1$. This is different to the behaviour of the one-player version of the game, where a fairly constant asymptotic density $p_\infty = 0.0287$ is reached at about $p = 0.15$ and remains at that level until approximately $p = 0.70$ [8]. Secondly, when periodic boundary conditions are used, the asymptotic density p_∞ increases by approximately 5% to 0.0381 compared to the situation of cutoff boundary conditions. Thirdly, it appears that the size of the lattice does not seem to affect the asymptotic density, although, as in the one-player version there may be small finite-size effects [9]. Finally, we estimated the ratio of the loser population over the winner population at the final state. A histogram of the results aggregated over 400 runs for a square lattice of size 100×100, where 100 runs were carried out for initial densities of 0.25, 0.5, 0.75 and 1.00, is shown in Figure 3. We observe that over 69% of the runs resulted in the ratio being over 0.5, i.e. the loser having more that one third of the final population.

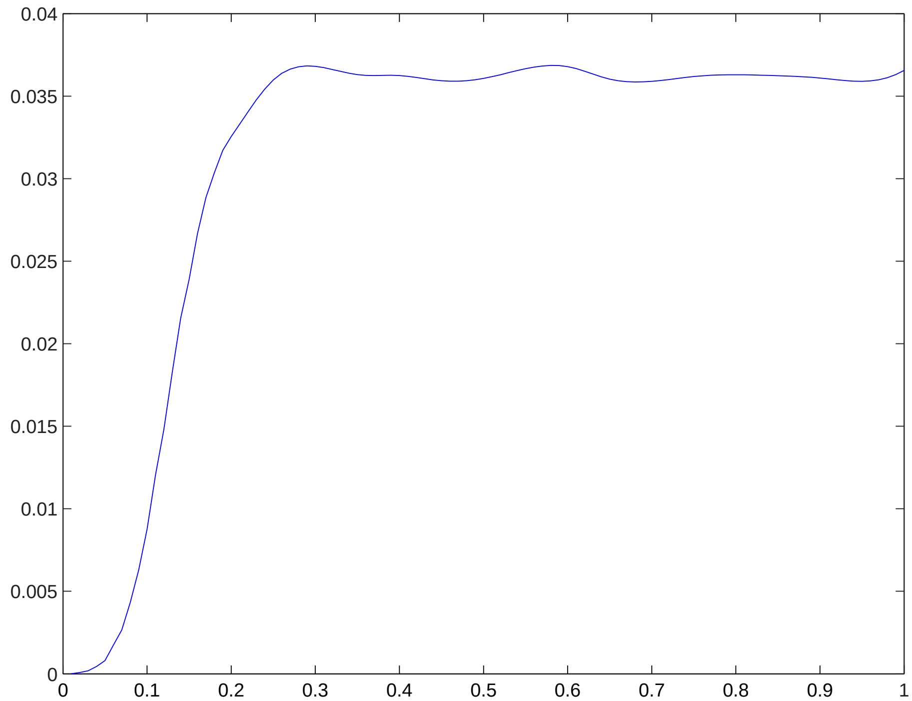

Figure 2: Plot of asymptotic density p_∞ against the initial density p

5 Concluding Remarks

Our main contribution is to have shown that, by injecting competitive elements into Conway's game of life, "Life" can be "interesting" when more than one player participates in the game. An applet demonstrating p2life can be accessed at

http://hades.dcs.bbk.ac.uk/users/gr/p2life/p2life.asp.

A problem that we are now investigating is how to convert p2life into a "real" game, i.e. where players are allowed to make moves between generations, which change the configuration of the tokens, and devise strategies to overpower or live side-by-side with their opponent.

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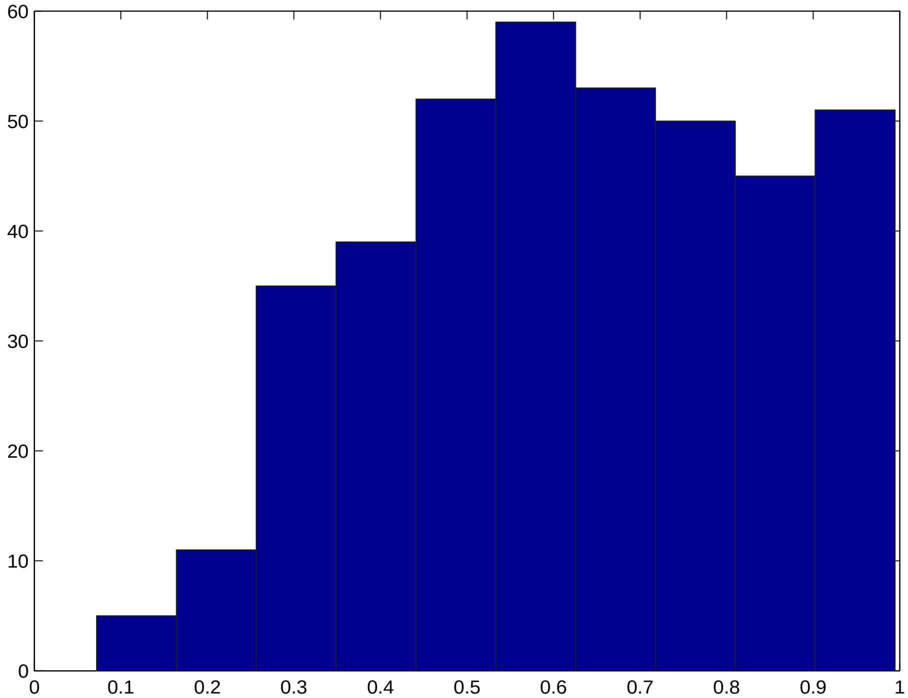

Figure 3: Histogram for the normalised difference between the winner and loser populations

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