# Canonicity Frameworks and Ultraproducts of Polarities

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The duality between modal algebras and Kripke frames depends on the formation of *canonical* extensions of algebras, a construction introduced in the 1940s by Jónsson and Tarski for Boolean algebras with additional operations preserving finite joins. After a hiatus of several decades, Gehrke and Harding [2] provided a theory of canonical extensions of lattice-ordered algebras in general. It builds a canonical extension as the lattice of all stable subsets of a polarity structure in the sense of Birkhoff.

This theory will be used to study a generalisation to lattice-ordered algebras of an influential result of Fine [1] from modal model theory. Fine's theorem states that if a modal logic is determined by a first-order definable class of frames, then it is valid in all its canonical frames. That was generalised in [3] to the fact that the complex algebras of an ultraproducts-closed class of relational structures generates a variety of Boolean algebras with operators that is closed under canonical extensions.

The talk will describe an axiomatisation of the structural relationships underlying this result by providing a list of conditions on an algebra/structure duality that suffice to ensure that if a class of polarity-based structures is closed under ultraproducts, then the stable set lattices of these structures generate a variety of algebras that is closed under canonical extensions.

In particular, situations to which this applies arise when the polarity-based structures have stable set lattices whose additional operations are first-order definable over the structures and preserve all joins, or all meets. That includes the residuated operations modelling the fusion and implication connectives of a number of substructural logics. A partial account of this work is given in [4].

## References

[1] Fine, K., *Some connections between elementary and modal logic*, in: S. Kanger, editor, *Proceedings of the Third Scandinavian Logic Symposium*, North-Holland, 1975 pp. 15–31.

[2] Gehrke, M. and J. Harding, *Bounded lattice expansions*, Journal of Algebra **239** (2001), pp. 345–371.

[3] Goldblatt, R., *Varieties of complex algebras*, Annals of Pure and Applied Logic **44** (1989), pp. 173–242.

[4] Goldblatt, R., *Canonical extensions and ultraproducts of polarities*, arXiv:1709.09798 (2017).