An Example of Bayesian Inference

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There's a disease that is affecting 1% of the population. We have a test for the disease that's only somewhat accurate: 80% of people with the disease will get a positive result. Unfortunately, 9.6% of people without the disease will also get a positive result. We apply the test to you and it turns out positive. What is the probability that you actually have the disease?

// Prior Probability

$P(D=1) = 0.01$ $P(D=0) = 0.99$

// Likelihood

$P(+|D=1) = 0.8$ $P(+|D=0) = 0.096$

$$
\begin{aligned}
P(+) &= P(+,D=1) + P(+,D=0) && \text{// Partition Rule} \\
&= P(+|D=1)\, P(D=1) + P(+|D=0)\, P(D=0) && \text{// Chain Rule} \\
&= 0.8*0.01 + 0.096*0.99 \\
&\approx 0.103
\end{aligned}
$$

// Posterior Probability computation using the Bayes' Rule

$$
\begin{aligned}
P(D=1|+) &= P(+,D=1) / P(+) && \text{// Chain Rule} \\
&= P(+|D=1)\, P(D=1) / P(+) && \text{// Chain Rule} \\
&= 0.8*0.01 / 0.103 \\
&\approx 0.078 = 7.8\%
\end{aligned}
$$

P.S.

Consider a population of 10,000 persons.
Of those, 100 will have the disease, and 80 will test positive.
That leaves 10000-100 who won't have the disease, or 9900.
The number of folks who don't have the disease but tested positive is 9.6% of 9900, or 950.
80 / (950 + 80) = 0.078 or 7.8%.